Towards a Unified Method to Implement Transit-Time Effects in Pi-Topology HBT Compact Models

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Abstract—**Four different approaches to include transit-time effects into Π-topology HBT equivalent circuits are investigated in order to assess their compatibility with the physics-based T topology. The aim is to find an implementation that not only yields an exact model but also has a unique set of parameters in both Π and T case. This is of prime importance for reliable parameter extraction and thus physical significance of the model. It is achieved using a transcapacitance approach. The theoretical considerations are supported by a practical example comparing measured and modeled HBT behaviour.**

I. Introduction

Although GaAs-based HBTs nowadays are widely available in standard MMIC technology, circuit designers are still waiting for a commonly available standard model for circuit design. But not only the large-signal model, even the method how to determine the small-signal equivalent circuit parameters is still under discussion in the respective publications.

One finds that most papers dealing with small-signal parameter-extraction techniques rely on T-circuit topology, usually because it is "more physical". On the other hand, it is attractive to maintain backward compatibility with the standard SPICE Gummel-Poon model, at least for the isothermal DC case. This requires compact large-signal models to be formulated in Π-topology. However, both formulations are not equivalent in all cases. It depends on the implementation of transit time that is associated with the current source in the model. This starting point leads to a well-known paradoxon: One may be able to determine the small-signal parameters quite accurately, but those parameters cannot directly be translated into large-signal parameters. The extracted physically significant parameters become mere starting points for a global optimization of the large-signal model. Obviously, this is also a problem when developing a compact model from measurement data.

Although the reported large-signal models yield good results that match measured data very well, we will stress the question of compatibility with T-topology for the following reasons. Firstly, it does not seem to be of any advantage to abandon the T-description with its physical meaning completely and to switch to Π-topology in small-signal considerations, too. Secondly, reliability of the parameter extraction procedure is most important, since it determines the basic accuracy of the model. A simplified model with a stable parameter-extraction routine is always preferable to a sophisticated one that leaves one alone with a huge number of unknowns.

This paper reconsiders four different methods how to introduce transit time into Π-topology large-signal models. The aim is to find the formulation for transit times in Π-topology that describes the extracted T-topology data most accurately. The investigation will be restricted to the small-signal case where a single parameter τ is used to account for transit-time effects. In large-signal models, this parameter will be split into different functions representing the different physical origin and bias dependence of the parts of the total transit time. Also superpositions of the implementations discussed below may be used.

II. T-Topology Parameters Derived from Π-Topology Parameters

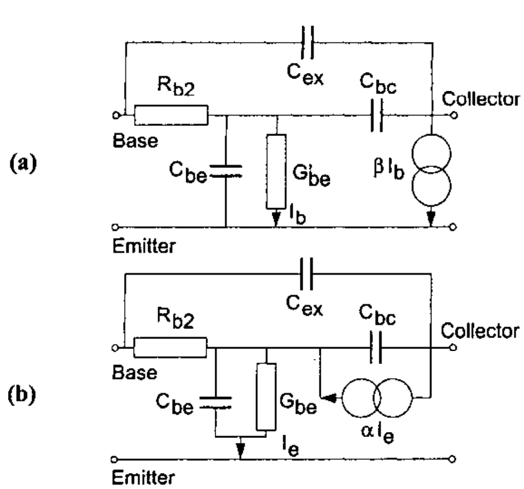

Fig. 1. Intrinsic equivalent circuits of HBT. (a) Π-topology, (b) T-topology. Extrinsic elements not shown.

In this section, the parameters of the T-topology equivalent circuit Fig. 1b will be derived from the Π-topology circuit Fig. 1a. Besides the different location of the current source, also the driving current is different in the two cases, as indicated in the respective figures. Calculation of the T-topology base-emitter admittance Y_{be} and α from Π-topology yields:

$$Y_{be} = j\omega C_{be} + G'_{be}(1+\beta) \quad (1)$$

$$\alpha = \frac{\beta}{(1+\beta) + j\omega(C_{be}/G'_{be})}$$

It is obvious that an arbitrary choice of the frequency dependence of β will result in a complex, frequency-dependent G_{be}, C_{be}, and α. Only if β has a constant real value β_0, Π- and T-topology are equivalent:

$$Y_{be} = j\omega C_{be} + G_{be} \quad (2)$$

$$\alpha = \alpha_0 \frac{1}{1 + j\omega/\omega_\alpha} \quad (3)$$

With $\omega_\alpha = 1/(C_{be}/G_{be})$, $G_{be} = G'_{be}(1+\beta_0)$ and $\alpha_0 = \beta_0/(1+\beta_0)$.

However, in the microwave range, transit times cannot be neglected. And, instead of (3), it also is usual to model the T-topology current source by:

$$\alpha = \frac{\alpha_0\, e^{-j\omega\tau}}{1 + j\omega/\omega_\alpha} \quad (4)$$

In the following, four different implementations of transit time in the Π-circuit will be investigated. These are

- use C_{be} to tune ω_α properly
- apply an excess-phase network
- introduce a time delay: $\beta = \beta_0\, e^{-j\omega\tau}$
- a base-collector transcapacitance

Since it is desirable that a model yields physically meaningful results for arbitrarily chosen positive parameters as (4) does, it is useful to consider the impact of the different transit-time implementations on the loci of α. It is not assured in all cases, that α shows a low-pass behaviour, for $C_{be} > 0$. While the overall behaviour of the equivalent circuit still might yield HBT-like characteristics, the unexpected frequency dependence, especially for large values of τ, may cause confusion.

A. Manipulation of ω_α

This method is the simplest way to introduce a time delay. It does not need enhancements in circuit topology. Hence, T- and Π-scheme are equivalent. In this case, C_{be} is modified in order to change the time constant of the base-emitter admittance to $1/\omega'_\alpha \overset{!}{=} 1/\omega_\alpha + \tau$ in order to use eqn. (3) as a first-order approximation of eq. (4) [1]. Thereby, one sacrifices input return loss for the description of the current source.

B. Excess-phase network

In order to gain an additional degree of freedom compared to the previous implementation, an excess-phase network, Fig. 2, may be introduced. In this case, the current I'_b is used as driving current of the current source. β thereby gets a bessel-filter-like frequency dependence instead of a single-pole low-pass, and the network easily can be implemented in circuit-simulation software. The values of L' and C' are derived from a single time constant: $L' = \tau/3$, $C' = \tau$ [2], [3]. However, this implementation can not directly be trans-

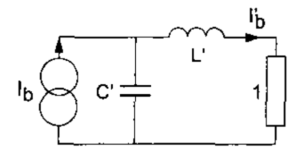

Fig. 2. Excess-phase network

lated into T-topology. In contrast, the T-topology parameters have the following form:

$$Y_{be} = j\omega C_{be} + G'_{be} + \frac{\beta_0 G'_{be}}{1 + j\,\omega\tau - \omega^2\tau^2/3} \quad (5)$$

$$\alpha = \beta_0 \frac{1}{1 + j\,\omega\tau - \omega^2\tau^2/3} \cdot \frac{G'_{be}}{Y_{be}} \quad (6)$$

Although α has a low-pass characteristic, it might exceed the initial value α_0 for lower frequencies. If C_{be} is assumed to be neglibly small, $|\alpha| < \alpha_0$ will only hold for frequencies $\omega\tau > \sqrt{\beta_0 - 1/2}$. In the other case, the inequality

$$\tau \le \frac{\sqrt{(3\beta_0+3)(3\beta_0-1)} - 3\beta_0}{(2\beta_0 - 1)G'_{be}/C_{be}} \quad (7)$$

must be satisfied.

For high frequencies, real(Y_{be}) approaches G_{be} due to the $-\omega^2$ term in the denominator. Besides that reduction, real(Y_{be}) might swing into the negative region.

C. Time delay

Introducing a time delay in the form $\beta = \beta_0 e^{-j\omega\tau}$ is problematic in a large-signal model. Since those models usually have to be formulated in the time domain, a previous time step $(t-\tau)$ has to be accessed. This is not always possible, or, e.g. in case of user-compiled models in Agilent's ADS, only a constant τ is allowed and a bias-dependence cannot be considered. Besides that, the base-emitter admittance cannot be described by frequency-independent parameters [4]. Calculating T-topology parameters from this equivalent circuit yields:

$$Y_{be} = j\omega C_{be} + G'_{be}\,(1 + \beta_0 e^{-j\omega\tau}) \quad (8)$$

$$\alpha = \frac{\beta_0\, e^{-j\omega\tau}}{j\omega C_{be}/G'_{be} + 1 + \beta_0\, e^{-j\omega\tau}} \quad (9)$$

In this formulation, low-pass behaviour is caused only by C_{be}. In the worst case, $C_{be} \to 0$, α will oscillate and describe circles from α_0 to the maximum point $\beta_0/(\beta_0 - 1)$

for $\omega\tau = (2n-1)\cdot\pi$. Therefore, it is necessary that τ is small enough compared to C_{be}/G'_{be}. More precisely, in order to ensure a low-pass behaviour, one has to choose

$$\tau \leq \frac{\sqrt{1/\alpha_0}-1}{G'_{be}/C_{be}} \qquad (10)$$

D. Transcapacitance

In case of the transcapacitance-approach [5]–[7], a charge $Q_b = f(V_{be})$ is inserted in parallel with C_{bc}. This quantity represents the charge stored in base and collector. This leads to a transcapacitance, i.e. a capacitance that is driven by the voltage of another branch. The transcapacitance is introduced automatically, when C_{bc} is considered to be a function of collector current [8]–[12]. In general, transcapacitances are dangerous since they act like voltage-driven current sources with a weighting factor $j\omega$, and thereby may cause excessive gain at high frequencies. In HBTs, however, it turns out, that a transcapacitance C_{tr} modifies α as follows:

$$\alpha = \frac{\alpha_0 - j\omega(C_{tr}/G_{be})}{1+j\omega(C_{be}/G_{be})} \qquad (11)$$

In order to assure $\alpha \leq \alpha_0$, it is necessary to set $C_{tr} \leq C_{be}\alpha_0$. In case of this formulation, the transit time is modeled indirectly. α is described by eq. (3), and thereby T– and Π-topology are equivalent.

III. Practical Example: State-of-the-Art HBT

In this section, we will apply the previous theoretical considerations to a practical example, a $3\times30\,\mu\text{m}^2$ GaInP/GaAs-HBT fabricated at the Ferdinand-Braun-Institut [13] on the 4″ process line. As an example, a typical bias point, $V_{ce} = 3\,V$, $I_c = 18\,\text{mA}$, $f_t = 28\,\text{GHz}$ is chosen. For this HBT, maximum $f_t = 38\,\text{GHz}$ is achieved at $I_c = 35\,\text{mA}$. The elements of the small-signal equivalent-circuit in T-topology are determined by an analytical algorithm [14]. In this bias point, the S-parameters are well modeled by the T-topology equivalent circuit with $\tau = 2.46\,\text{ps}$ and $\omega_\alpha = 2\pi\,55\,\text{GHz}$. In a first crude approach, these parameters are directly inserted into the different Π-topology circuits. For the transcapacitance approach, $C_{tr} = \tau G_{be}$ is chosen.

First, the frequency dependence of the T-topology current source α is calculated from the different Π-circuits. Fig. 3 provides the results. For the original T-circuit, α is modeled well up to f_t (curve marked 0). For the approaches with modified ω_α (1) and the transcapacitance term (4), one finds good agreement of $\angle\alpha$ with the measurements as well, while $|\alpha|$ is underestimated by (1) and overestimated by (4).

The extracted values lead to $|\alpha|$ increasing beyond unity in case of the excess-phase network (2) and time-delay (3) approaches, which in turn both underestimate $\angle\alpha$. This is due to the fact that both eqn. (7) and (10) are not satisfied.

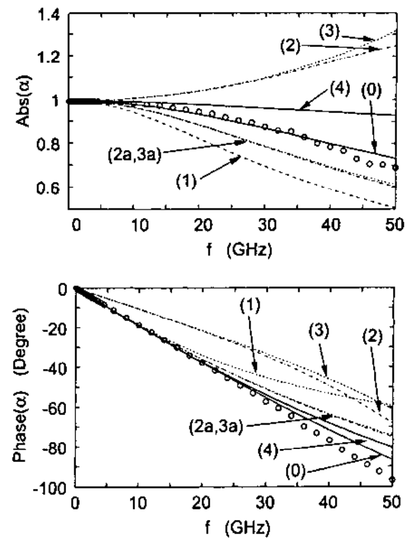

Fig. 3. Magnitude and phase of α. Measured (symbols) and modeled (lines) data with: (0) T-topology, (1) modified ω_α, (2) excess-phase network, (2a) the same with modified parameters, (3) time delay, (3a) the same with modified parameters, (4) transcapacitance.

In order to obtain better agreement with the measured data, $\tau = 1.1\,\text{ps}$ and $\omega_\alpha = 2\pi\,30\,\text{GHz}$ are chosen in both cases. These parameters yield low-pass behaviour for $|\alpha|$ and fit the $\angle\alpha$ very well. These curves are marked as (2a,3a) in Fig. 3.

The S-parameters, Fig. 4 and 5, are modeled well by the original T-circuit (curve marked 0), the approach with modified ω_α (1), and the transcapacitance term (4), as expected from the previous investigation of α. The non-physical behaviour of α modeled by the excess phase and time delay approaches lead to overestimated $|S_{21}|$, and underestimated $|S_{22}|$ and $\angle S_{22}$. The results differ from the desired curves, but no obvious non-physical behaviour is observed. The S-parameters also are modeled well for these two approaches if modified parameters are used (curves marked as 2a,3a).

While it is necessary to extract new parameters from the extracted τ and ω_α values in case of the excess-phase network and time-delay approaches, the model works directly in case of the modulated ω_α and transcapacitance approaches. However, the modulation of ω_α is a simplifed approach that leads to deviations mainly with respect to S_{22} beginning at frequencies around 10 GHz or one third of the transit frequency. The transcapacitance approach, on the other hand, directly yields an almost perfect fit.

IV. Conclusions

Four common methods to implement transit times into compact Π-topology HBT models are investigated. The leading question was how to keep the physical significant

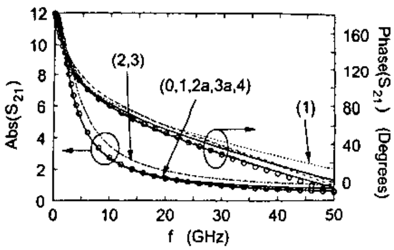

Fig. 4. Magnitude and Phase of S_{21}. Measured (symbols) and modeled (lines) with: (0) T-topology, (1) modified ω_α, (2) excess-phase network, (2a) the same with modified parameters, (3) time delay, (3a) the same with modified parameters, (4) transcapacitance.

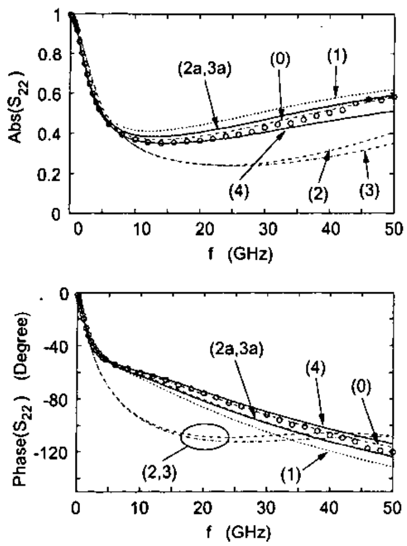

Fig. 5. Magnitude and phase of S_{22}. Measured (symbols) and modeled (lines) with: (0) T-topology, (1) modified ω_α, (2) excess-phase network, (2a) the same with modified parameters, (3) time delay, (3a) the same with modified parameters, (4) transcapacitance.

parameters of the T-topology equivalent circuit in the Π-topology. The following conclusions can be drawn:

Neglecting the transit time can partly be compensated by properly adjusting the base-emitter pn-junctions time constant. However, this is possible only in the lower frequency range, i.e., below $f_t/3$ in the present example.

If transit times are modeled by introducing a time delay or by using an excess-phase network in the Π-topology, even reasonably chosen parameters may lead to non-physical (and therefore unexpected) behaviour in terms of the corresponding T-topology. Especially current gain α may exceed unity at high frequencies. In order to prevent this, the time constant has to be set within the bounds defined in this paper. The other drawback of these implementations is that parameters extracted for the T-topology equivalent circuit cannot be directly transferred into Π-topology.

The transcapacitance approach, therefore, turns out to be the most promising approach to implement transit times into compact HBT models. It yields accurate results and preserves physical significance of the model parameters of the T-topology, even when Π-topology is used.

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